LESSON 5. 2a

Properties of Rational Exponents and Radicals

Today you will:

- Use properties of rational exponents to simplify expressions with rational exponents.
- Use properties of radicals to simplify and write radical expressions in simplest form.
- Practice using English to describe math processes and equations

Core Vocabulary:

- simplest form of a radical, p. 245
- conjugate, p. 246
- like radicals, p. 246

Previous:

- properties of integer exponents
- rationalizing the denominator
- absolute value

Properties of Rational Exponents

The properties of integer exponents that you have previously learned can also be applied to rational exponents.

Core Concept

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \dfrac{1}{a^m}, a \neq 0$	$36^{-1/2} = \dfrac{1}{36^{1/2}} = \dfrac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\dfrac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, b \neq 0$	$\left(\dfrac{27}{64}\right)^{1/3} = \dfrac{27^{1/3}}{64^{1/3}} = \dfrac{3}{4}$

Use the properties of rational exponents to simplify each expression.

a. $7^{1/4} \bullet 7^{1/2} = 7^{(1/4 + 1/2)} = 7^{3/4}$

b. $(6^{1/2} \bullet 4^{1/3})^2 = (6^{1/2})^2 \bullet (4^{1/3})^2 = 6^{(1/2 \bullet 2)} \bullet 4^{(1/3 \bullet 2)} = 6^1 \bullet 4^{2/3} = 6 \bullet 4^{2/3}$

c. $(4^5 \bullet 3^5)^{-1/5} = [(4 \bullet 3)^5]^{-1/5} = (12^5)^{-1/5} = 12^{[5 \bullet (-1/5)]} = 12^{-1} = \dfrac{1}{12}$

d. $\dfrac{5}{5^{1/3}} = \dfrac{5^1}{5^{1/3}} = 5^{(1 - 1/3)} = 5^{2/3}$

e. $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\dfrac{42}{6}\right)^{1/3}\right]^2 = (7^{1/3})^2 = 7^{(1/3 \bullet 2)} = 7^{2/3}$

Simplifying Radical Expressions

The Power of a Product and Power of a Quotient properties can be expressed using radical notation when $m = \frac{1}{n}$ for some integer n greater than 1.

Core Concept

Properties of Radicals

Let a and b be real numbers such that the indicated roots are real numbers, and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}},\ b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

Use the properties of radicals to simplify each expression.

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6$

Product Property of Radicals

b. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\dfrac{80}{5}} = \sqrt[4]{16} = 2$

Quotient Property of Radicals

An expression involving a radical with index n is in ***simplest form*** when these three conditions are met:

- Reduce/simply the number inside the radical (no radicands have perfect nth powers as factors other than 1).
- No fractions inside the radical (no radicands contain fractions).
- No radicals appear in the denominator of a fraction.

Rationalize the denominator!!!

- Multiply the expression by an appropriate form of 1 that eliminates the radical from the denominator.

Write the expressions (a) $\sqrt[3]{135}$ and (b) $\dfrac{\sqrt[5]{7}}{\sqrt[5]{8}}$ in simplest form.

SOLUTION

a. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5}$ Factor out perfect cube.

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$ Product Property of Radicals

$= 3\sqrt[3]{5}$ Simplify.

b. $\dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} = \dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \dfrac{\sqrt[5]{4}}{\sqrt[5]{4}}$ Make the radicand in the denominator a perfect fifth power.

$= \dfrac{\sqrt[5]{28}}{\sqrt[5]{32}}$ Product Property of Radicals

$= \dfrac{\sqrt[5]{28}}{2}$ Simplify.

For a denominator that is a sum or difference involving square roots,

Example: $\frac{1}{5+\sqrt{3}}$

multiply both the numerator and denominator by the ***conjugate*** of the denominator.

The expressions

$a\sqrt{-b} + c\sqrt{-d}$ and $a\sqrt{-b} - c\sqrt{-d}$

are conjugates of each other, where a, b, c, and d are rational numbers.

So for the above example the conjugates are:

$\frac{1}{5+\sqrt{3}}$ and $\frac{1}{5-\sqrt{3}}$

Write $\frac{1}{5+\sqrt{3}}$ in simplest form.

SOLUTION

Remember: for a denominator that is a sum or difference involving square roots, multiply both the numerator and denominator by the conjugate of the denominator.

$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \cdot \frac{5-\sqrt{3}}{5-\sqrt{3}}$$

The conjugate of $5+\sqrt{3}$ is $5-\sqrt{3}$.

$$= \frac{1(5-\sqrt{3})}{5^2-(\sqrt{3})^2}$$

Sum and Difference Pattern

$$= \frac{5-\sqrt{3}}{22}$$

Simplify.

Vocab: ***Like Radicals***

Radical expressions with the same index and radicand are like radicals.

To add or subtract like radicals, use the Distributive Property.

Simplify each expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10}$

b. $2(8^{1/5}) + 10(8^{1/5})$

c. $\sqrt[3]{54} + \sqrt[3]{2}$

SOLUTION

a. $\sqrt[4]{10} + 7\sqrt[4]{10} = (1 + 7)\sqrt[4]{10} = 8\sqrt[4]{10}$

b. $2(8^{1/5}) + 10(8^{1/5}) = (2 + 10)(8^{1/5}) = 12(8^{1/5})$

c. $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3 - 1)\sqrt[3]{2} = 2\sqrt[3]{2}$

Homework

Pg 248, #3-47 (do at least half)